Individual Choices in a Non–Consequentialist Framework: The Case of Procedures

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by

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1 Motivation and Some Examples

It is probably true to say that most of welfare economics is consequentialist in nature. In other words, what really matters in analyses of welfare are outcomes in terms of individual and social welfare. However, as Suzumura and Xu (2001) write, "there do exist people who care not only about welfaristic features of the consequences, but also about non–welfaristic features of the consequences or even non–consequential features of the decision–making procedure through which these consequences are brought about" (p. 424). Within a firm, for example, most employees will probably prefer that major organisational changes be carried out after some general discussion that involves the staff and not at the bidding of the board of directors. Within the political sphere, many people will most likely prefer that a new policy be brought about through public debate and not clandestinely. A quite different example concerns the last Presidential elections in the US where many people felt rather uneasy about the way in which the current President came out as the winner. The electoral process and the management of the election were of wider concern.

In bargaining theory and expected utility theory analogous phenomena exist. Let E_1 and E_2 be two economic environments that yield identical utility–possibility sets, i.e. $S(E_1) = S(E_2)$. If f stands for a particular bargaining solution, we obtain $f(S(E_1)) = f(S(E_2))$ as a fundamental principle of bargaining analysis. Both information on how the underlying commodity allocations came about and information on how the set of feasible utility allocations was arrived at, are aspects that do not matter under f. One could, again, argue that these features should matter, at least under certain circumstances. In expected utility theory à la von Neumann–Morgenstern, a one–stage lottery and a multi–stage lottery are judged to be equivalent utility–wise as long as the outcomes and the net cumulative probability of each outcome are the same.

Consider another case where non–consequential aspects seem to matter. Judy is celebrating her birthday for which her mother has made a beautiful cake to be enjoyed during the afternoon when Judy is surrounded by all her friends. One particular slice is especially gorgeous . It *is* Judy's piece. In variant a of our story, Judy's younger brother whom she adores takes a few slices of the cake while all the girls are out in the garden. The particularly gorgeous piece is left untouched. The girls come back inside, see what has happened, laugh about Judy's clever brother and thoroughly enjoy the rest of the cake, with Judy taking the slice meant for her. In variant b, Judy's elder brother with whom she is constantly quarreling sneaks into the room, pinches a few pieces without touching Judy's slice and is about to leave when his sister comes in. Judy bursts into tears and screams that she will not take even one slice of her cake any more. Under variant a, Judy would have chosen her slice under S (the full cake) and she still chooses this slice under $S' \subset S$. Under variant b, she would have picked her slice under S but takes no slice at all from S'. Obviously, the shrinking of the cake seems to matter. Let us call the shrinkage from S to S' a procedure. Obviously, in our story, there are two distinct procedures i and j, and if $C(\cdot)$ stands for a choice function, we obtain $C([S', i]) = \{x\}$ and $C([S', j]) = \emptyset$, if x represents Judy 's slice and S' denotes the slices that remain under procedures i and j (we assume that the slices taken away were exactly the same in both cases).

Coming back to bargaining theory, to Nash's (1950) solution in particular, his condition of independence of irrelevant alternatives requires that if the solution for a certain set of utility allocations S is still possible or feasible for some subset $S' \subseteq S$, then this solution should also resolve the bargaining problem for S'. In other words, this particular shrinkage of the bargaining set itself is of no relevance.

In the following sections, we wish to discuss non–consequential features of decision–making, procedural aspects in particular. We shall try to develop a theoretical frame that, we hope, will be able to deal with these characteristics appropriately.

2 More Examples and Some Structure

Traditional choice theory does not consider procedural aspects or the latter are deemed insignificant for choice. Let there be a finite set of alternatives X and strict subsets $A \subset X$. Furthermore, let N be a finite set of distinct procedures (concerning production of the objects or their availability or both). If procedures do not matter, we get

$$C([A, i]) = C([A, j]) \neq \emptyset$$

for all $A \subset X$ and any $i, j \in N, i \neq j$. This result could be called *consequentialist*, since, obviously, the only thing which is of relevance is the set of objects A (and not the aspect how A came about). *Non–consequentialism* would hold if for at least one $A \subset X$, one would have $C([A, i]) \neq C([A, j])$ for $i \neq j$, or for at least one $A \subset X$ and for at least one procedure $i \in N$, $C([A, i]) = \emptyset$.

At this point, we wish to briefly discuss a situation that is a modification of an example given by Sen (1988). Let us consider a country with a finite set of newspapers, some highly political, others only marginally so. Among the first, there is one daily paper which is considered to be the government's mouthpiece, but with undeniable editorial qualities nevertheless. The person we consider chooses this paper among all the available papers. Let us now imagine that for some reason, the government decides to ban all papers except for its mouthpiece. The person we observe now decides to refrain from choosing. In other words, her choice set with respect to papers is the empty set. So if n is the mouthpiece and k stands for the procedural aspect of a government ban, we would have $C([\{n\}, k]) = \emptyset, n \in S$, where S denotes the set of newspapers before government intervention. It seems plausible to argue that n would still be chosen by the individual if it was left as the only available newspaper after a major natural disaster l, i.e. $C([\{n\}, l]) = \{n\}$. This indicates that again procedural aspects seem to matter and, furthermore, that bits of information about how a certain alternative fares "choice–wise" under other procedures could be of relevance as well.

One realizes that once such a procedural path is taken, one can go into various directions. One of the directions would be to argue that an alternative x, for which

$C([\{x\},i]) = \emptyset$ for some $i \in N$, should *never* be picked under the same i. In other words, for any $A \subset X$ such that $x \in A$, $x \notin C([A],i)$. One could say, however, that this is a too strict requirement so that if there is at least one other alternative y available under procedure i for which it is not true that $C([\{y\},i]) = \emptyset$, then x would be choosable. This argument could be modified again by saying that an alternative x for which $x \notin C([\{x\},i])$, would need a "sufficient or minimal number" of other objects around in order to be chosen. In our example above, one could reason that if a couple of other papers were left intact under the ban (i.e. under the given procedure i), mouthpiece n would perhaps still be picked.

How can these different aspects be formalized? Given X and N, as defined above, let K be the set of all non–empty subsets of X and $K \times N$ be the cartesian product of K and N. The elements of $K \times N$ will be denoted by $[A,i], [B,j]$, etc. The intended interpretation of $[A,i]$ is that the subset A is brought about or produced by the procedure i. A choice function C is a mapping from $K \times N$ to $K \cup \{\emptyset\}$ such that for all $A \in K$, all $i \in N$, $C([A,i]) \subseteq A$. Note that we shall allow $C([A,i]) = \emptyset$ for some $A \in K$. The emptiness of a choice set can be regarded as an inaction. The individual considered refuses to choose anything from the set of choosable elements. It should be clear from our discussions so far that we interpret $C([A,i])$ as follows: given a subset A, which is brought about or produced by a procedure i, $C([A,i]) \subset A$ is the set of alternatives that the individual would like to pick from A given i. Let R be a weak ordering (reflexive, transitive and complete) over X. For all $A \in K$, define $\hat{C}(A,R) = \{x \in A | xRy$ for all $y \in A\}$. Furthermore, we define $\hat{C}(\emptyset, R) = \emptyset$.

As a first step, we concentrate on those elements x in A such that the choice set of the singleton $\{x\}$ under procedure i is empty.

Definition 1. For all $i \in N$, all $x, y \in X$, x and y are *similar* with respect to i if $C([\{x\},i]) = C([\{y\},i])$.

In other words, x is similar to itself and for distinct $x, y \in X$, if the choice sets for $\{x\}$ and for $\{y\}$ under procedure i are empty, then x and y are similar with respect to i.

Definition 2. Given a choice function C, for all $i \in N$, all $A \in K$, define $W(A,i) := \{x \in A | C([\{x\},i]) = \emptyset\}$.

$W(A,i)$ consists of all those elements x in A that under procedure i are not chosen from the singleton set. We now propose two ways in which elements of $W(A,i)$ can be "treated" when the set of choosable elements A contains more than a single element.

Definition 3. A choice function C is *LP-rationalizable* iff for all $i \in N$, there exists an ordering R_i over X such that for all $A \in K$, $C([A,i]) = \hat{C}(A - W(A,i), R_i)$ if $W(A,i) = A$ and $C([A,i]) = \hat{C}(A, R_i)$ if $W(A,i) \neq A$.

Definition 3 says that elements from $W(A,i)$ are potential candidates for choice as long as at least one other alternative outside of $W(\cdot)$ is available. This is a requirement that expresses a relatively mild protest. One may want to demand more. In our newspaper example above, we spoke about a "sufficient or minimal" number of other objects that should be potentially available to the choosing individual. Therefore, we now introduce the concept of critical sets under a given procedure i.

For all $i \in N$ and all $x \in X$, let $\Omega(x,i)$ be defined as the set of all elements in K such that x is chosen from $[B,i]$, but x is not chosen from $[B-\{y\},i]$ for some y in B and $y \neq x$, and let $M(x,i)$ be defined as the universal set X when x is not chosen from $[B,i]$ for all B in K, and as the set of all elements in $\Omega(x,i)$ such that each of them contains the smallest number of alternatives when x is chosen from $[E,i]$ for some E in K. The elements in $M(x,i)$ will be denoted by $m(x,i), m'(x,i)$, etc. To illustrate, consider the following example. Let $X = \{x,y,z\}$ and $N = \{i\}$, and the choice function C be defined as follows: $C([\{x\},i]) = \emptyset, C([\{y\},i]) = \{y\}, C([\{z\},i]) = \emptyset, C([\{x,y\},i]) = \{x\}, C([\{x,z\},i]) = \emptyset, C([\{y,z\},i]) = \{y\}, C([\{x,y,z\},i]) = \{x\}$. Then, $\Omega(x,i) = \{\{x,y,z\},\{x,y\}\}, \Omega(y,i) = \emptyset$, $\Omega(z,i) = \emptyset$, $M(x,i) = \{\{x,y\}\}$, $M(y,i) = \emptyset$ and $M(z,i) = \{\{x,y,z\}\}$.

Therefore, for all $x \in X$ and all $i \in N$, elements in $\Omega(x,i)$ are to be interpreted as critical sets for x under i, and elements in $M(x,i)$ are those critical sets that have the smallest cardinalities in $\Omega(x,i)$ if x is chosen from at least one subset E of X that contains x and $M(x,i)$ is the universal set if x is not chosen from any subset B of X containing x. In our newspaper example, it is easy to imagine that individuals will differ with respect to what they consider as a minimally sufficient variety of newspapers. The minimally required number of other options will most likely depend on the properties of alternative x itself. If x were a rather apolitical paper, the critical–set–requirement would probably look quite different. The minimally required number of other options may also depend on which other options are available. Entertainment magazines wouldn't really add much to the variety of political opinions.

Based on the notion of critical sets, we introduce the following definition.

Definition 4. A choice function C is *CP-rationalizable* iff for all $i \in N$, there exists an ordering R_i over X such that $\forall A \in K, C([A,i]) = \hat{C}(A - \Pi(A), R_i)$, where $\Pi(A) = \{x \in A | \#A < \# m(x,i), m(x,i) \in M(x,i)\}$.

3 Axioms and First Results

In the sequel we shall introduce various axioms that will allow us to characterize choice behaviour when procedural aspects matter. The first two axioms reflect the two types of protest that we were discussing in the previous section. Axiom WPC expresses the idea that if two options x and y are not picked from singleton sets, given procedure i, then if one of them is not chosen from set $A \subset X$, it continues not to be picked if the other option is added to set A. Axiom LPC formulates the cardinality requirement for elements belonging to $W(A,i)$.

Weak Protest Consistency (WPC): For all i in N, all A in K, all x in A and all y in X, if x and y are similar with respect to i, then x is not chosen from $[A \cup \{y\}, i]$ whenever x is not chosen from $[A,i]$.

Limited Protest Consistency (LPC): For all i in N, all A in K, all x in X, if x is not chosen from $[\{x\},i]$ and $(A \cup \{x\})$ contains fewer alternatives than $m(x,i)$, then x is not chosen from $[A \cup \{x\}, i]$, where $m(x,i)$ is in $M(x,i)$.

The next axiom is rather innocuous. It guarantees the non–emptiness of the choice set, given set A and procedure i, if there exists at least one element that does not belong to $W(A, i)$.

Non-empty Choice of No Protest Situations (NCNP): For all i in N, all A in K, if there exist x, y in A such that x and y are not similar with respect to i, then $C([A, i]) \neq \emptyset$.

The following four axioms are consistency requirements when the set of choosable options expands or contracts. The first two axioms deal with the case of set contraction. They are straightforward modifications of Arrow's (1959) rationality condition and Sen's (1977) α-condition or Nash's (1950) independence condition, respectively. Sen's (1977) set–expansion condition β is used here without any modification. The final consistency condition CEC is natural, once empty choice sets are admitted.

Restricted Arrow Condition (RAC): For all i in N, all A and A' in K, if $A' \subseteq A$ and there exists x in A' such that the choice from $[\{x\}, i]$ is not empty, then $C([A', i]) = A' \cap C([A, i])$ whenever $C([A, i]) \cap A' \neq \emptyset$.

Restricted Contraction (α^r): For all i in N, all A and B in K, if $x \in B \subseteq A$ and x is chosen from $[A, i]$, then x is chosen from $[B, i]$ whenever B contains at least as many elements as $m(x, i)$, where $m(x, i)$ is in $M(x, i)$.

Expansion (β): For all i in N, all A and B in K, if x and y are in $B \subseteq A$, and x and y are both chosen from $[B, i]$, then x is chosen from $[A, i]$ if and only if y is chosen from $[A, i]$.

Consistency of Empty Choice (CEC): For all i in N, all A and B in K, if B is a subset of A and the choice from $[A, i]$ is empty, then the choice from $[B, i]$ is empty.

At the beginning of section 2, we asserted that traditional choice theory does not consider procedural aspects. This statement can be turned around by saying that conventional choice theory defines conditions of expansion and contraction consistency independently of the underlying procedure. In Arrow's rationality condition from 1959, for example, it does not matter at all whether the transition from superset A to subset A', let's say, was caused by procedure i or procedure j or some other process, nor would the consistency requirement be affected in any way if the shrinkage from A to A' were generated by procedure i and the transition from A' to A'' were caused by some other procedure j. In other words, the traditional consistency conditions are stronger than our requirements since they are implicitly defined for all i, j in N.

The first two results we wish to offer are the following. Their proofs can be found in the Appendix.

Theorem 1. A choice function C is LP-rationalizable iff it satisfies axioms WPC, NCNP and RAC.

Theorem 2. A choice function C is CP-rationalizable iff it satisfies LPC, CEC, β and α^r.

4 Extensions – Part 1

So far, we have confined our theoretical analysis to cases where only one procedure at a time is considered. However, as argued above, it may be important for a person to ask whether a certain alternative would, perhaps, not be chosen under several other procedures or even not be picked under any procedure at all. In our newspaper example, we argued that an individual may refuse to pick the only paper left after government has introduced a ban on all other political papers. Our individual may also decide not to buy the only political paper left if the other papers have been bought up by the government and turned into completely apolitical ones. However, as indicated before, the individual would, perhaps, continue to acquire the government's mouthpiece if the other papers had been destroyed by a major disaster or went out of business due to financial mismanagement. In the cake example, Judy would have continued to pick her slice if some of the pieces had been destroyed by a big jug that accidentally fell on the birthday cake. In other words, we now introduce some degree of interdependence among different procedures that permits additional information to enter the process of choice.

In line with the foregoing arguments, we first want to define, for each $A \in K$, the set of elements in A each of which is not chosen from the singleton set under any procedure at all. So we define $\Gamma(A) = \{x \in A | C([\{x\}, i]) = \emptyset \text{ for all } i \in N\}$. Later on, we shall say that elements belonging to $\Gamma(A)$ should, with some justification, be deleted from choice. However, one can argue that this is a rather weak requirement. One may demand something more severe. Let us consider the following refinement. For all elements $x \in X$, we define the set of procedures $N(x)$ with the property that x is not picked from the singleton set. More formally, for all $x \in X$, $N(x) = \{i \in N | C([\{x\}, i]) = \emptyset\}$. Next, the individual is supposed to compare the cardinality of $N(x)$ with some x–specific threshold level $q(x) > 0$ which defines a level of tolerance for the choosing individual. In other words, if $\#N(x)$ is larger than $q(x)$, $x \in A$ has "failed sufficiently" under various procedures and will be deleted from further choice. For all $A \in K$, $\Gamma'(A)$ collects all those elements from A that have this property. So we obtain $\Gamma'(A) = \{x \in A | \#N(x) > q(x)\}$. It is obvious that the introduction of the threshold level $q(x)$ only makes sense if the procedures that we consider can be clearly defined. Our introductory examples hopefully demonstrated that this is what we are having in mind. So in our newspaper example, the government ban on other papers, a government buy–out of other papers, financial mismanagement among the other papers leading to their bankruptcy are instances of clearly defined procedures.

Using the newly introduced notions above, we can now define the concept of LP' rationalizability.

Definition 5. A choice function C is *LP'-rationalizable* iff, $\forall i \in N$, there exists an ordering R_i over X such that, $\forall A \in K$, $C([A, i]) = \hat{C}(A - W(A, i), R_i)$ if $W(A, i) = A$ and $C([A, i]) = \hat{C}(A - \Gamma'(A), R_i)$ if $W(A, i) \neq A$.

Furthermore we define

Protest Consistency Based on a Threshold (TPC): For all $i \in N$, all $x \in X$ and all $A \in K$, if x is not chosen from $[\{x\}, i]$ and $N(x)$ contains more elements than $q(x)$, then x is not chosen from $[A \cup \{x\}, i]$.

This condition says that an alternative that is rejected from the singleton set under a sufficient number of procedures will never be chosen when the set of choosable elements expands, under any procedure i.

We can now formulate the following result whose proof is also given in the Appendix.

Theorem 3. A choice function C is LP'-rationalizable iff it satisfies TPC, WPC, NCNP and RAC.

The idea laid out above that an alternative $x \in A \subseteq X$ should be eliminated from further choice when it has failed sufficiently under various procedures, can be weakened. The weakest version of this view uses $\Gamma(A)$ for all $A \in K$ that we defined at the beginning of this section. Then only those options are eliminated from further choice that would not be picked from the singleton set under all procedures possible.

Using $\Gamma(A)$ instead of $\Gamma'(A)$ for all $A \in K$, we can redefine condition TPC in a straightforward manner to obtain TPC*. Also, the notion of LP'-rationalizable has to be modified by replacing $\Gamma'(A)$ in its definition by $\Gamma(A)$ to arrive at LP^*-rationalizability. Doing this, we obtain the following corollary to Theorem 3.

Corollary 3. A choice function C is LP^*-rationalizable iff it satisfies TPC*, WPC, NCNP, and RAC.

What we have worked out above, viz. using the newly defined set $\Gamma'(A)$ for all $A \in K$ and changing LP-rationalizability into the notion of LP'-rationalizable, can also be done with respect to the concept of minimally critical sets. Instead of finding the set of all $A \in \Omega(x,i)$ such that $\#A \leq \#B$, $\forall B \in \Omega(x,i)$, we have to define the set of all $A \in \Omega(x,i)$ such that $\#(A - \Gamma'(A)) \leq \#(B - \Gamma'(B))$, $\forall B \in \Omega(x,i)$. In other words, the cardinality requirement now focuses on augmented sets after all elements in $\Gamma'(\cdot)$ have been discarded from further consideration. What we finally arrive at, but we abstain from spelling out the details, is an analogue to Theorem 2.

5 Extensions – Part 2

Up to now, we focused on procedures or processes that shrink the universal set X of all conceivable alternatives to particular feasible subsets $S, S', \ldots$ of X. We did not discuss the aspect of production methods as such that give rise to alternatives contained in a feasible set and moreover, we did not look at the interaction between those production methods and procedures or processes that shrink the set X to alternative feasible subsets of X. We now turn to these issues and present some analysis.

Let us describe the following choice situations faced by an individual in a given society. To begin with, as far as the physical commodity, *coal*, from the set S of feasible alternatives is concerned, we describe it more broadly and consider alternatives of the following kind: *coal mined by men*, *coal mined by women* and *coal mined by children*. Let us discuss the following two situations. In situation I, the feasible set contains two alternatives: *coal mined by women* and *coal mined by children*, and the availability of these alternatives is due to emergency conditions (for example, due to a natural catastrophe where men have to fulfil some other tasks involving greater risks and are no longer available to do coal mining). In situation II, the feasible set contains the same two alternatives as before, i.e. *coal mined by women* and *coal mined by children*, but the alternatives become available under normal circumstances. It is conceivable that the individual under consideration may consider the alternative *coal mined by women* acceptable in situation I, while the same alternative is deemed unacceptable in situation II. It is therefore suggested that the individual's choice behaviour in these two situations may differ. The difference in the individual's choice behaviour seems to result both from how the physical commodity *coal* is being produced and the interaction between the production method and the procedure that narrows down the initial set of alternatives to some proper subset.

To formalize the above intuition, we first specify a commodity or an alternative in detail. Following Debreu (1959)[1], we specify production methods that generate physical commodities or alternatives. Therefore, let T be the set of all relevant production methods. We assume that T can be partitioned into three non–empty subsets T^1, T^2 and T^3, with the interpretation that T^1 consists of those production methods that are *never* acceptable[2], T^2 comprises those production methods that are *always* acceptable, and T^3 consists of those methods whose acceptability is contingent *on other things*. Consequently, a commodity or an alternative from the universal set X henceforth specifies the underlying production method in an explicit form. For example, we now distinguish between coal mined by men and coal mined by women (or children), carpets woven by women or by children, or road construction done by men or by women. For the ease of presentation, we assume that $X = O \times T$, where O stands for the set of all features of commodities or alternatives other than T. As a consequence, alternatives in X can be denoted by $(x_0, \tau), (y_0, \tau')$, etc., where $x_0, y_0 \in O$ and $\tau, \tau' \in T$.

Next, we focus on possible procedures that give rise to a feasible subset from a given set of commodities or alternatives. For this purpose, let N stand for the set of procedures that narrow down a given set S to some proper subsets of S. We assume that N can be partitioned into procedures N^1 that we describe as interventionist man–made procedures (such as dictatorship, wars, and other kinds of imposed interferences and obstructions) and N^2 that we characterize as both interventionist but "not directly" man–made procedures (natural catastrophes, nuclear accidents) and non–interventionist procedures such as the market system or a given legal system. We assume that neither N^1 nor N^2 is empty and that they completely exhaust N, i.e., $N = N^1 \cup N^2$.

We now wish to consider a relationship between procedures that determine the availability of certain subsets and those generated subsets themselves. With K being the set of *all* non–empty subsets of X, we distinguish between elements from N^1 generating $K^1 \subset K$ and elements from N^2 generating $K^2 \subset K$, where normally $K^1 \cap K^2 \neq \emptyset$. In general, it may not be possible to generate all elements of K from either N^1 alone or from N^2 alone. Also, an interventionist procedure such as dictatorship may bring about subsets from K that the market system or a legal system which belong to N^2 may not be able to effectuate. Given the distinction between N^1 and N^2, it should be clear that all elements of the cartesian product $K^1 \times N^1$ and all elements of the cartesian product $K^2 \times N^2$ are potentially feasible.

[1]Note that in his "Theory of Value", Debreu (1959) was careful in defining a commodity. He also emphasized the aspect of production when he said that "wheat available now and wheat available in a week play entirely different economic roles for a flour mill which is to use them" (p. 29).

[2]As we shall see later on, this can be endogenously defined by saying that commodities produced under elements from T^1 will never be picked by a choosing individual (see axiom A.1. below).

Given our definition of X from above, we consider $A \subset O$, $\tau \in T$ and $i \in N$. Then $(A \times \{\tau\})$, $(A' \times \{\tau'\})$ are subsets of X, where $\tau, \tau' \in T$ are the underlying production methods. These subsets can be generated by shrinking procedures from either N^1 or N^2. So we may, for example, have triples of the form $(A \times \{\tau\}; i)$ with $i \in N^1$ and $(A' \times \{\tau'\}; j)$ with $j \in N^2$, where subset $(A \times \{\tau\})$ belongs to K^1 and $(A' \times \{\tau'\})$ belongs to K^2. A choice function chooses nothing or picks an alternative $(a, \tau) \in (A \times \{\tau\})$ from $(A \times \{\tau\}; i)$. For example, we may have the following: $C(\{\text{coal mined by women}\}; j) = \emptyset$ and $C(\{\text{coal mined by women}\}; i) = \{\text{coal mined by women}\}$, where j stands for a non–interventionist procedure from N^2 while $i \in N^1$ represents an imposed interference such as a case of war.

Analogous to previous discussions, we may wish to propose some notion of rationalizability of choice functions in this extended framework. To begin with, for a given choice function C, all $i \in N$, all $A \subseteq O$, all $x \in O$, and all $\tau \in T$, let

$$W'(A, \tau; i) := \{(a, \tau) \in A \times \{\tau\} | C([\{a\} \times \{\tau\}; i]) = \emptyset\},$$

and

$$N'(x, \tau) := \{i \in N | C([\{x\} \times \{\tau\}; i]) = \emptyset\},$$

and

$$\Gamma''(A, \tau) := \{(x, \tau) \in A \times \{\tau\} | \# N'(x, \tau) > k\};$$

where k is some positive integer. Then, consider the following definition:

Definition 6. A choice function C is rationalizable on the basis of an *interaction between production and procedure* (IPP-rationalizable) iff, $\forall i \in N$, there exists an ordering R_i over X such that, $\forall A \subseteq O$, $\forall \tau \in T$:

$$\begin{aligned} C([A \times \{\tau\}; i]) &= \hat{C}(A - W'(A, \tau; i), R_i) & \text{if } \tau \in T^1, \\ C([A \times \{\tau\}; i]) &= \hat{C}(A, R_i) & \text{if } \tau \in T^2, \\ C([A \times \{\tau\}; i]) &= \hat{C}(A - \Gamma''(A, \tau), R_i) & \text{if } \tau \in T^3. \end{aligned}$$

To give a flavor of the above notion of IPP-rationalizability of a choice function, we consider the following properties that are satisfied by such a choice function.

The first axiom is rather uncontroversial and can be stated as follows.

A.1. For all $(A \times \{\tau\}; i)$ from either $K^1 \times N^1$ or $K^2 \times N^2$, if $\tau \in T^1$ and $(a, \tau) \in A \times \{\tau\}$, then $(a, \tau) \notin C([A \times \{\tau\}; i])$.

Once a commodity or option has been produced by a production method from T^1, it will never be chosen, independent of whether the feasible set of elements was generated by N^1 or N^2. In terms of eliminating possible objects of choice, this axiom states a requirement that is lexicographic in relation to other criteria.

The following axiom deals with interventionist man–made procedures, where the variety of choice becomes an issue.

A.2 For all $\tau \in T^3$, if $(A \times \{\tau\}; i) \in K^1 \times N^1$ with $\#(A \times \{\tau\}) \leq k$, k being some positive integer and $(a, \tau) \in A \times \{\tau\}$, then $(a, \tau) \notin C([A \times \{\tau\}; i])$

A.2 expresses the idea that under procedures from N^1, the decision maker is sensitive to limitations of choice. So choices from sets with a cardinality of k (or smaller) will be rejected.

The following axiom considers production methods from T^2.

A.3. For all $\tau \in T^2$, and all $(A \times \{\tau\}; i) \in X \times N$, $C([A \times \{\tau\}; i]) \neq \emptyset$.

Obviously, the production methods $\tau \in T^2$ constitute the standard case of choice theory.

The point we wished to make in this section is the following. If we proceed with our analysis in this extended framework and consider the interactions between the production methods that generate physical commodities or alternatives and the procedural aspects that give rise to the feasible sets of physical commodities or alternatives, we can formulate appropriate contraction and expansion consistency properties and explore how the notion of IPP-rationalizability of a choice function can be characterized axiomatically. However, in this paper we abstain from pursuing the derivation of formal results along these lines any further.

6 Concluding Remarks

In various decision and choice situations, an individual may have good reasons to consider non-consequential features of decision problems, procedural aspects in particular. This non-consequential consideration of decision problems is certainly different from the standard approach in the theory of rational choice within economics and calls for an unconventional investigation into an individual's choice behaviour. This paper has proposed a theoretical framework that enables us to analyze and examine the related issues of non-consequentialism reflected in an individual's choices.

In our theoretical framework, we have analyzed several types of choice behaviour that cannot be rationalized within the standard approach. A common thread in this non-standard approach is the novel notion that an individual may choose to pick nothing from feasible sets of alternatives if those feasible sets are brought about in particular ways that are deemed unacceptable. This emptiness of a choice set is in sharp contrast to the often assumed non-emptiness of a choice set in the standard framework and reflects the procedural concern of the individual under consideration: if the way that gives rise to various feasible subsets is "not good" according to the individual's subscribed view, the individual may register a protest by refusing to choose any alternative from the given feasible subset even though some alternatives in the set would have a positive value for this person. Depending on the context, we have proposed various notions of rationalizability of a choice function and have characterized some of them axiomatically.

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Appendix

Proof of Theorem 1. It can be checked that if a choice function C is LP–rationalizable, then it satisfies axioms WPC, NCNP and RAC. Therefore, we have only to show that if a choice function C satisfies WPC, NCNP and RAC, then it is LP–rationalizable.

Let C be the choice function that satisfies WPC, NCNP and RAC. Let $i \in N$. First, we note that for all $A \in K$, if $C([\{a\}, i]) = \emptyset$ for all $a \in A$, then, by WPC, $C([A, i]) = \emptyset$. Note also that the above observation is independent of any underlying binary relations. If $C([\{x\}, i]) = \emptyset$ for all $x \in X$, then define the binary relation R_i over X as follows: xI_iy for all $x, y \in X$. Clearly, given WPC, the R_i thus defined is LP–rationalizable. Now, suppose that for some $x \in X$, $C([\{x\}, i]) \neq \emptyset$. Given NCNP, $C([X, i]) \neq \emptyset$. We first define $X_1, X_2, \cdots, X_k$ as follows: $X_1 = C([X, i]), X_2 = C([X - X_1, i]), \cdots, X_k = C([X - \cup_{j=1}^{k-1} X_j, i])$ and k is such that $W(X_k, i) \subseteq X_k$, $C([X_k, i]) \neq \emptyset$ and $C([X - \cup_{j=1}^{k} X_j, i]) = \emptyset$. Now, define the binary relation R_i as follows: for all $x, y \in X, [xR_iy$ iff $x \in X_m$ and $y \in X_n$ where $m \leq n]$. Clearly, R_i as defined above is reflexive, transitive and complete. Next, we show that C is LP-rationalizable; that is, for all $A \in K$, $C([A, i]) = \hat{C}(A - W(A, i), R_i)$ if $W(A, i) = A$ and $C([A, i]) = \hat{C}(A, R_i)$ if $W(A, i) \neq A$.

Consider $A \in K$ such that $W(A, i) = A$. Let $A = \{a_1, \cdots, a_p\}$. Since $W(A, i) = A$, for all $a \in A, C([\{a\}, i]) = \emptyset$. In particular, $C([\{a_1\}, i]) = \emptyset$. Then, by the repeated use of WPC, we must have $C([A, i]) = \emptyset$. On the other hand, $\hat{C}(A - W(A, i), R_i) = \hat{C}(\emptyset, R_i) = \emptyset$. Therefore, in this case, $C([A, i]) = \hat{C}(A - W(A, i), R_i)$.

Now, consider $A \in K$ such that $W(A, i) \neq A$. Clearly, there exists $x \in A$ such that $C([\{x\}, i]) = \{x\}$. It is then clear that $W(X, i) \neq X$. From the definition of R_i, clearly, $C([X, i]) = \hat{C}(X, R_i)$. It is also clear that, from the definition of R_i, $C([X_j, i]) = \hat{C}(X_j, R_i)$ for all $j = 1, \cdots, k$. We now show that $\hat{C}(A, R_i) = C([A, i])$ in this case.
(1) Let $a \in C([A, i])$. By RAC, noting that $C([\{x\}, i]) = \{x\}$, it must be true that $a \in C([\{a, x, y\}, i])$ for all $y \in A$. Suppose to the contrary that $a \notin \hat{C}(A, R_i)$. Then, from the definition of R_i, there exists $b \in A$ such that $b \in X_p$ and $a \in X_q$ with $p < q$. Note that $C([X_p, i]) = \hat{C}(X_p, R_i)$. We must have $b \in C([X_p, i])$. Consider $C([\{a, b, x\}, i])$. If xP_ib, then $x \in C([X_m, i])$ where $m < p$. By RAC, $\{x\} = C([\{x, a, b\}, i])$, a contradiction. If xI_ib, then $x \in C([X_p, i])$. By RAC, $\{x, b\} = C([\{x, a, b\}, i])$, a contradiction. If bP_ix, then, by RAC, $\{b\} = C([\{x, a, b\}, i])$, another contradiction. Hence, $a \in \hat{C}(A, R_i)$.
(2) Let $a \in \hat{C}(A, R_i)$. Then, from the definition of R_i, for all $y \in A$, aR_iy. Let X_p be such that $a \in C([X_p, i])$. Then, noting that $C([\{x\}, i]) = \{x\}$, by RAC, we must have $a \in C([A, i])$. Therefore, C is LP-rationalizable. ■

Proof of Theorem 2. It can be checked that if a choice function C is CP–rationalizable, then it satisfies axioms LPC, CEC, β and α^r. Therefore, we have only to show that if a choice function C satisfies LPC, CEC, β and α^r, then it is CP–rationalizable.

Let C be the choice function that satisfies LPC, CEC, β and α^r. Let $i \in N$. We distinguish two cases: case (i) $C([X, i]) = \emptyset$ and case (ii) $C([X, i]) \neq \emptyset$. In case (i), by CEC, $C([A, i]) = \emptyset$ for all $A \in K$. Therefore, $M(x, i) = X$ for all $x \in X$. Define the binary relation R_i over X as follows: xI_iy for all $x, y \in X$. Clearly, R_i is reflexive, transitive and complete. It is also clear that from the definition of $\Pi(A)$, $\Pi(A) = A$, so $\hat{C}(A - \Pi(A), R_i) = \emptyset = C([A, i])$. Hence, C is CP-rationalizable. In case (ii), define $X_1, X_2, \cdots, X_k$ as follows: $X_1 = C([X, i]), X_2 = C([X - X_1, i]), \cdots, X_{k-1} = C([X -$

$\cup_{m=1}^{k-2} X_m, i]), X_k = X - \cup_{m=1}^{k-1} X_m$ where k is such that $X_{k-1} \neq \emptyset$, $X - \cup_{m=1}^{k-1} X_m \neq \emptyset$, and $[C([X - \cup_{m=1}^{k-1} X_m, i]) = X - \cup_{m=1}^{k-1} X_m$ or $C([X - \cup_{m=1}^{k-1} X_m, i]) = \emptyset]$. Now, define the binary relation R_i over X as follows: xR_iy iff $x \in X_m$ and $y \in X_n$ where $m \leq n$. From the construction, R_i is reflexive, transitive and complete. Next we show that $C([A, i]) = \hat{C}(A - \Pi(A), R_i)$ for all $A \in K$. Let $A \in K$. Let $x \in C([A, i])$. Given that $x \in C([A, i])$, clearly, $\#m(x, i) < \#A$. Hence, $x \notin \Pi(A)$. By β and α^r, we are back to the classical case. From the definition of R_i, xR_ia for all $a \in A$. Therefore, $x \in \hat{C}(A - \Pi(A), R_i)$. That is, $C([A, i]) \subseteq \hat{C}(A - \Pi(A), R_i)$. Suppose next that $x \in \hat{C}(A - \Pi(A), R_i)$. From the definition of $\Pi(A)$ and by LPC, for all $z \in \Pi(A)$, $z \notin C([A, i])$. From the definition of R_i, there exist m, n with $m \leq n$ such that $x \in X_m$ and for all $y \in A - \Pi(A)$, $y \in X_n$. Then, by α^r and β, $x \in C([A, i])$ follows from classical rationalizability. Hence, $\hat{C}(A - \Pi(A), R_i) \subseteq C([A, i])$. Therefore, $C([A, i]) = \hat{C}(A - \Pi(A), R_i)$. That is, C is CP-rationalizable. ■

Proof of Theorem 3. The necessity part of the theorem can be easily checked. We show the sufficiency.

Let C be the choice function that satisfies SPC′, WPC, NCNP, and RAC. Let $i \in N$. We distinguish two cases: case (i) $C([X, i]) = \emptyset$ and case (ii) $C([X, i]) \neq \emptyset$. In case (i), we define the binary relation R_i over X as follows: xI_iy for all $x, y \in X$. Then, by WPC, the R_i defined above is LP'-rationalizable. In case (ii), let $X' = X - \Gamma'(X)$. By SPC′ and NCNP, $C([X', i]) \neq \emptyset$. Define $X'_1, X'_2, \cdots, X'_k$ as follows: $X'_1 = C([X', i])$, $X'_2 = C([X' - X'_1, i]), \cdots, X'_k = C([X' - \cup_{j=1}^{k-1} X'_j, i])$ and k is such that $W(X'_k, i) \subseteq X'_k, C([X'_k, i]) \neq \emptyset$ and $C([X - \cup_{j=1}^{k} X'_j, i]) = \emptyset$. The remainder of the proof of the result is similar to that of Theorem 2 and we omit it. ■