

Measuring the Standard of Living: Uncertainty about Its Development*

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Abstract. Human development is about expanding the choices human beings have to lead lives that they value and is captured by its capability sets which consist of various functioning vectors. The standard of living is then reflected in capability sets. This paper proposes some particular ways of measuring the standard of living available to an agent, be it an individual or a whole country, when the direction of the development of society represented by a reference functioning vector is uncertain. We provide axiomatic characterizations of the proposed measures.

Keywords: Functioning, Capability, Uncertainty, Social Progress, Standard of Living

JEL Classification Numbers: C78, D63, D71, D74

1 Introduction

Income and wealth are important factors in order to provide and secure a decent standard of living. Economic growth may help to improve this situation. But human development is about much more. As the human development report 2001 asserts, development is about expanding the choices human beings have to lead lives that they value. Fundamental is “building human capabilities – the range of things that people can do or be in life” (Human Development Report (HDR), p. 9). And the report spells out the most basic capabilities for human development: To lead a long and healthy life, to be knowledgeable, and to have access to the resources needed for a decent standard of living.

The human development index aggregates these three basic dimensions of human development into a summary measure. As an indicator for a long and healthy life, life expectancy at birth is used from which a life expectancy index is constructed. An education index represents knowledge. The latter concept considers the adult literacy rate and a so-called gross enrolment ratio. From each of these an index is formed and then, the two indices are multiplied by factors of two-thirds and one-third respectively and additively combined to yield the education index. The logarithm of income is used to construct a GDP index that is supposed to capture the notion of a decent standard of living. All three indices thus formed are weighted by one-third and this weighted sum yields the human development index HDI.

The human poverty index for developing countries (HPI-1) and the poverty index for selected OECD countries (HPI-2) are constructed in a similar way. HPI-1, for example, is composed of three indices, each is weighted by the same exponent α and in their symmetric linear representation, each index is multiplied by one third (actually both HPI-1 and HPI-2 have the structure of a CES production function). In the HDR of 2001, a value of 3 is attached to the exponent α in order “to give additional but not overwhelming weight to areas of more acute deprivation” (HDR, p. 241).

There are various other indices of development which were developed recently. Their structure is roughly the same: they are multidimensional in character but eventually reduced to one numerical index. This reduction procedure involves an exercise in weighting as spelt out in our examples above. Clearly, a change of weights means affecting the aggregate outcome. In relation to the α -exponent, Anand and Sen (1997) admit that “there is an inescapable arbitrariness ” (p. 16) in its choice. Earlier on in their paper,

they are more explicit on this issue. “Since any choice of weights should be open to questioning and debating in public discussions, it is crucial that the judgments that are implicit in such weighting be made as clear and comprehensible as possible, and thus be open to public scrutiny” (p. 6).

The human development index is a handy tool without any doubt but as Sen, one of the originators of this index, emphasizes the choice of weights is a sensible issue and ultimately a matter for social choice based on valuational arguments (Sen, 2002, p. 7). Sen goes one step further and stresses the time dimension. “When the ingredients of a judgment are diverse, any aggregate index with *constant* weights (the emphasis is by the author) over its diverse constituent elements would tend to oversimplify the evaluative exercise” (p. 12). One has to be interested in the present situation of countries but sometimes, changes over time are of particular interest. The spread of diseases as well as a more restricted access to clean water resources are important for life expectancy in developing countries. So a higher weight for these aspects would signal particular attention. In more developed countries where death at an early age is no longer a pressing issue, social exclusion measured by long-term unemployment may justify a higher weight in future investigations. Therefore, departures from the current structure and usage of the various indices may seem legitimate.

In this paper, we propose a particular way of measuring the standard of living available to an agent as well as to a whole country. The agent or country will be characterized by a capability set consisting of various vectors of functionings possible at any given time. The basis for our theoretical analysis is Lancaster’s (1966) characteristics approach to consumer theory. In this approach consumer goods generate characteristics, and this is done according to a linear “input–output” relationship. The higher the income of a consumer or country, the higher are the maximally possible purchases of a particular good. However, in general, the consumer can choose among different consumer goods and, moreover, the consumer can spend part of his income on commodity a , let’s say, another part on good b , a third part on commodity c , etc. In other words, combinations of different commodities are possible and income-wise feasible. In the space of characteristics, we obtain, due to the linear “production technology”, star-shaped convex spaces.

In our context, we assume linear input–output relationships in a twofold way. Consumer goods (but also investment goods, like capital investments in land irrigation or education) generate characteristics and these characteristics lead to different functionings or functioning vectors. These represent health,

longevity, literacy and other basic qualities. Given a particular income (for an individual) or a particular budget (for a country), the individual (or country, respectively) can acquire various consumer goods (a country would, additionally, run different investment projects). These yield various functioning vectors and combinations of these generate convex spaces of functionings. These spaces span the agent's as well as a country's capability set. Due to the underlying linearity, they are star-shaped.

The human development index as well as the other indicators mentioned above produce a real number for each country under investigation. By doing so, a complete ordering over all countries concerned is generated. Both the ordering as well as measured differences in the HDI, for example, between two countries a and b reveal deficiencies. Among the countries with high human development, an HDI value in 1999 of 0.939 for Norway and a value of 0.831 for Slovakia show quite a large gap between the two countries, whereas Slovakia and Hungary seem to be at a very similar stage of human development, the latter's HDI index being 0.829 for the same year.

In this paper, we do not consider indices or real numbers as indicators or benchmarks for comparisons. As has become clear above, we shall focus on *vectors of functionings*. In order to be judged living a satisfactory life, an agent or a country must have a given functioning vector in her capability set. We readily admit that determining such a reference functioning vector is, conceptually speaking, not easy. For the moment, we wish to assume that this problem has been solved (we just refer to the development and refinement of the HDI and other indices over the last twelve years). To improve her standard of living in terms of functionings, given the uncertainty associated with the development of society (and the world economy), it is not immediately clear along which direction the agent's or the country's functioning vector will grow as time progresses. Furthermore – and now we come back to Sen's remarks on constant weights and the aspect of changes over time, the reference functioning vector may, and perhaps should, change over time in its composition, paying, perhaps, more attention to the access to clean water resources and adult illiteracy in developing countries, and, perhaps, paying more attention to long-term unemployment and youth unemployment in more developed countries. We investigate how the agent's or country's standard of living may be measured, given these uncertainties within and among societies. We shall also examine the case where the reference functioning vector lies outside the capability set of the agent or the country considered.

The structure of the paper is as follows. Section 2 introduces the basic

notation and definitions. Section 3 presents the axioms that we need for our first characterization result. Section 4 states this theorem and provides a proof. Section 5 introduces a deprivation–gap ordering and discusses a second result. The final section 6 is devoted to some concluding remarks.

2 Basic Notation and Definitions

Let \mathbb{R}_+^n be the non-negative orthant of the n -dimensional real space. The vectors in \mathbb{R}_+^n will be denoted by x, y, z, a, b, \dots , and are interpreted as functioning vectors (Sen (1985, 1987)). For all $x = (x_1, \dots, x_n), y = (y_1, \dots, y_n) \in \mathbb{R}_+^n$, define $x > y$ when $x_i \geq y_i$ for all $i = 1, \dots, n$ and $x_j > y_j$ for some $j \in \{1, \dots, n\}$, and $x \gg y$ when $x_i > y_i$ for all $i = 1, \dots, n$. For all $x, y \in \mathbb{R}_+^n$, we define the distance between them as follows: $\|x - y\| = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$.

At any given point of time, the set of all vectors that may be available to the individual is a subset of \mathbb{R}_+^n . Such a set will be called the individual's *capability set*. We will use A, B, C , etc. to denote the capability sets.

Our concern in this paper is to rank different capability sets in terms of the *standard of living* that they offer to the individual. In particular, we confine our attention to opportunity sets that are

(2.1) *compact*: a capability set $A \subseteq \mathbb{R}_+^n$ is compact iff A is closed and bounded,

(2.2) *convex*: a capability set $A \subseteq \mathbb{R}_+^n$ is convex iff, for all $x, y \in \mathbb{R}_+^n$ and all $\alpha \in [0, 1]$, if $x, y \in A$, then $\alpha x + (1 - \alpha)y \in A$,

(2.3) *star-shaped*: a capability set $A \subseteq \mathbb{R}_+^n$ is star-shaped iff, for all $x \in \mathbb{R}_+^n$ and all $t \in [0, 1]$, if $x \in A$, then $tx \in A$.

Let \mathcal{K} be the set of all capability sets that are compact, convex and star-shaped. For all $A, B \in \mathcal{K}$, we write $A \subseteq B$ for “ A being a subset of B ” and $A \subset B$ for “ A being a proper subset of B ”.

For all $A, B \in \mathcal{K}$ and all $x^* \in \mathbb{R}_+^n$, if [whenever $x^* \in B$, there is a neighborhood, $\mathcal{N}(x^*, \epsilon) = \{x \in \mathbb{R}_+^n : x \geq x^*, \|x - x^*\| \leq \epsilon\}$ where $\epsilon > 0$, of x^* such that $\mathcal{N}(x^*, \epsilon) \subseteq A$] and [for all $b \in B$ with $b > x^*$, there exists $a \in A$ such that $a \gg b$], then we say that B *lies entirely in A relative to x^** . Let $x^0 \in \mathbb{R}_+^n$ be the deprivation vector of functionings below which

the individual's standard of living is judged to be "poor". Throughout this paper, we assume that x^0 is fixed. For all $t > 0$, let

$$X(x^0, t) = \{x \in \mathbb{R}_+^n : x \geq x^0, \|x - x^0\| \leq t\}.$$

For all $A \in \mathcal{K}$, let

$$r(A) = \begin{cases} -1 & \text{if } x^0 \notin A \\ \max_t \{t \in \mathbb{R}_+ : \{x \in \mathbb{R}_+^n : x \geq x^0, \|x - x^0\| \leq t\} \subseteq A\} & \text{if } x^0 \in A \end{cases}$$

Figure 1 depicts the maximal $t \in \mathbb{R}_+$ for two capability sets A and B when $x^0 \in A \cap B$.

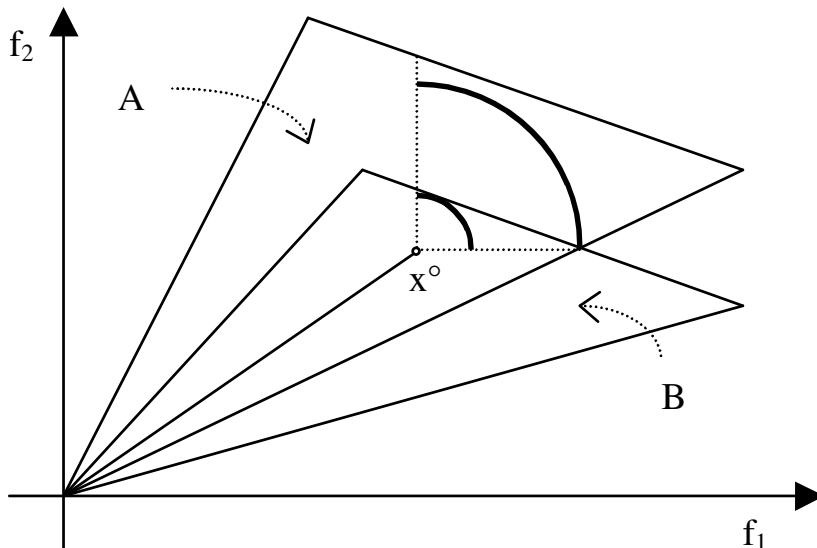


Figure 1: comparison of two capability sets A and B

Let \succeq be a binary relation over \mathcal{K} that satisfies *reflexivity*: [for all $A \in \mathcal{K}$, $A \succeq A$], *transitivity*: [for all $A, B, C \in \mathcal{K}$, if $A \succeq B$ and $B \succeq C$ then $A \succeq C$], and *completeness*: [for all $A, B \in \mathcal{K}$ with $A \neq B$, $A \succeq B$ or $B \succeq A$]. Thus, \succeq is an *ordering*. The intended interpretation of \succeq is the following:

for all $A, B \in \mathcal{K}$, $[A \succeq B]$ will be interpreted as “the degree of the standard of living offered by A is at least as great as the degree of the standard of living offered by B ”. \succ and \sim , respectively, are the asymmetric and symmetric part of \succeq .

3 Axiomatic Properties

In the following two sections, we present an axiomatic characterization of the standard of living ranking defined below:

$$\text{For all } A, B \in \mathcal{K}, A \succeq^r B \Leftrightarrow r(A) \geq r(B).$$

We begin by listing a set of axioms.

Definition 3.1. \succeq over \mathcal{K} satisfies

- (3.1.1) **Monotonicity** iff, for all $A, B \in \mathcal{K}$, if $B \subseteq A$ then $A \succeq B$.
- (3.1.2) **Betweenness** iff, for all $A, B \in \mathcal{K}$, if $A \succ B$ with $x^0 \in A \cap B$, then there exists $C \in \mathcal{K}$ such that B lies entirely in C relative to x^0 and $A \succ C \succ B$.
- (3.1.3) **Dominance** iff, for all $A, B \in \mathcal{K}$, if $x^0 \notin B$, then $A \succeq B$, and furthermore, if $x^0 \in A$, then $A \succ B$.
- (3.1.4) **Domination in Terms of Uncertain Development** iff, for all $A, B \in \mathcal{K}$, if there exists $t > 0$ such that $X(x^0, t) \cap A = X(x^0, t)$, and $B \cap X(x^0, t) \subset X(x^0, t)$, then $A \succ B$.

The intuition behind Monotonicity is simple and easy to explain. It requires that whenever B is a subset of A , then A is ranked at least as high as B concerning standards of living offered. Betweenness requires that when A is judged to offer a higher standard of living than B relative to the deprivation vector x^0 , there must exist a set C such that B lies entirely in C and A offers a higher standard of living than C , which in turn offers a higher standard of living than B . Dominance requires that whenever the deprivation vector x^0 is not achievable in B , the standard of living offered by B cannot be higher than that offered by any other capability set A , and furthermore, if the deprivation vector x^0 is achievable under A , then A offers a higher standard of living than B . Domination in Terms of Uncertain Development requires

that, for two capability sets A and B , whenever A results from progress made in all dimensions of functioning vectors, while B does not offer this particular kind of progress, the standard of living under A is judged to be higher than that offered by B .

4 A First Characterization Result

Theorem 4.1. Suppose \succeq over \mathcal{K} is an ordering. Then, \succeq satisfies Monotonicity, Betweenness, Dominance, and Domination in Terms of Uncertain Development if and only if $\succeq = \succeq^r$.

Proof. It can be checked that \succeq^r is an ordering and satisfies Monotonicity, Betweenness, Dominance and Domination in Terms of Uncertain Development. We now show that if \succeq over \mathcal{K} satisfies Monotonicity, Betweenness, Dominance and Domination in Terms of Uncertain Development, then $\succeq = \succeq^r$.

- (i) We first show that, for all $t > 0$ and all $A, B \in \mathcal{K}$, if $r(A) = t = r(B)$ and $B \cap X(x^0, t) = X(x^0, t) = A \cap X(x^0, t)$, then $A \sim B$. Suppose $A \succ B$. Then, by Betweenness, there exists $C \in \mathcal{K}$ such that B lies entirely in C relative to x^0 , and $A \succ C \succ B$. Since $r(B) = t > 0$, B lies entirely in C relative to x^0 , B and C are compact and star-shaped, and for some positive $t' > t$ and some set $C' \in \mathcal{K}$, $\{x \in \mathbb{R}_+^n : x \geq x^0, x \in C'\} = X(x^0, t')$, $C' \subseteq C$, and $B \cap X(x^0, t') \subset X(x^0, t')$. By Monotonicity, $C \succeq C'$ and by Domination in Terms of Uncertain Development (henceforth, Domination for short), $C' \succ B$. Hence, $A \succ C' \succ B$. Noting that $r(A) = t < t' = r(C')$, we must have $X(x^0, t) \cap C'$ is a proper subset of $C' \cap X(x^0, t') = X(x^0, t')$. By Domination, since $A \cap X(x^0, t') \subset X(x^0, t')$, we obtain $C' \succ A$ which is in contradiction to $A \succ C'$. Therefore, it is not true that $A \succ B$. Similarly, it can be shown that it is not true $B \succ A$. Therefore, $A \sim B$.
- (ii) Second, we show that for all $A, B \in \mathcal{K}$, if $r(A) > r(B) > 0$, then $A \succ B$. Let $A, B \in \mathcal{K}$ be such that $r(A) > r(B) > 0$. Consider $A', B' \in \mathcal{K}$ such that $A' \cap X(x^0, r(A)) = X(x^0, r(A))$, $B' \cap X(x^0, r(B)) = X(x^0, r(B))$ and $B' \subset A'$. Since $r(A) > r(B) > 0$, such A' and B' exist. From (i), $A' \sim A$ and $B' \sim B$. By Domination, $A' \succ B'$. Then, $A \succ B$ follows from transitivity of \succeq .

- (iii) Third, we show that for all $A, B \in \mathcal{K}$, if $r(A) > 0 = r(B)$, then $A \succ B$. Note that, since $r(A) > 0 = r(B)$, it must be true that $B \cap X(x^0, r(A)) \subset A \cap X(x^0, r(A)) = X(x^0, r(A))$. By Domination, $A \succ B$ follows easily.
- (iv) We next show that, for all $A, B \in \mathcal{K}$, if $x^0 \notin A$ and $x^0 \notin B$, then $A \sim B$. Since $x^0 \notin A$, by Dominance, $B \succeq A$. Similarly, by Dominance, from $x^0 \notin B$, it follows that $A \succeq B$. Therefore, $A \sim B$.
- (v) We now show that for all $A, B \in \mathcal{K}$, if $B = \{x \in \mathbb{R}_+^n : x = tx^0, \forall t \in [0, 1]\}$, $x^0 \in A$ and $r(A) = 0$, then $A \succ B$ for $t < 1$ and $A \sim B$ for $t = 1$. For the first case, since $x^0 \in A$ and $x^0 \notin B$, we obtain, by Dominance, that $A \succ B$. Consider now the case that $t = 1$. Then $x^0 \in B$ and $x^0 \in A \cap B$. First, since $A \in \mathcal{K}$, clearly, $B \subseteq A$. By Monotonicity, $A \succeq B$. Suppose that $A \succ B$. Then, by Betweenness, there exists $C \in \mathcal{K}$ such that B lies entirely in C relative to x^0 and $A \succ C \succ B$. Note that, since B lies entirely in C relative to x^0 and $x^0 \in B$, $r(C) > 0$. From (iii) above and $r(A) = 0$, $C \succ A$ follows immediately, which is a contradiction to $A \succ C$ obtained earlier. Therefore, $A \sim B$.
- (vi) From (v), for all $A, B \in \mathcal{K}$, if $x^0 \in A \cap B$ and $r(A) = r(B) = 0$, then $A \sim B$ follows immediately.
- (vii) To complete the proof, we note that, for all $A, B \in \mathcal{K}$, if $x^0 \in A$ and $x^0 \notin B$ and $r(A) = 0$, by Dominance, $A \succ B$.

Therefore, (i) – (vii), together with the transitivity of \succeq complete the proof of Theorem 4.1. ■

5 Further Properties and A Deprivation-Gap Ordering

The ordering \succeq^r defined in Section 3 and characterized in Section 4 ranks all capability sets $A, B \in \mathcal{K}$ with $x^0 \notin (A \cup B)$ equally in terms of the standard of living. This is rather unsatisfactory. In this section, we first propose a ranking rule that avoids this undesirable feature. We then propose several properties to characterize this new ranking rule.

To begin with, we define the notion of a capability set B lying entirely in a capability set A . For all $A, B \in \mathcal{K}$, if [for all $b \in B$ with $b \gg 0$, there exists a neighborhood $\mathcal{N}(b, \epsilon) = \{x \in \mathbb{R}_+^n : x \geq b, \|x - b\| \leq \epsilon\}$ where $\epsilon > 0$ of b such that $\mathcal{N}(b, \epsilon) \subseteq A$] and [for all $b \in B$, there exists $a \in A$ such that $a \gg b$], then B is said to lie entirely in A . Note that whenever B lies entirely in A and $x^0 \in A \cap B$, then B lies entirely in A relative to x^0 .

For all $t > 0$ and all $x^0 \in \mathbb{R}_+^n$, let $O(x^0, t) = \{x \in \mathbb{R}_+^n : \|x - x^0\| \leq t\}$.

For all $A \in \mathcal{K}$, let

$$r^*(A) = \begin{cases} -\min_t \{t \in \mathbb{R}_+ : \{x \in \mathbb{R}_+^n : \|x - x^0\| \leq t\} \cap A \neq \emptyset\} & \text{if } x^0 \notin A \\ \max_t \{t \in \mathbb{R}_+ : \{x \in \mathbb{R}_+^n : x \geq x^0, \|x - x^0\| \leq t\} \subseteq A\} & \text{if } x^0 \in A \end{cases}$$

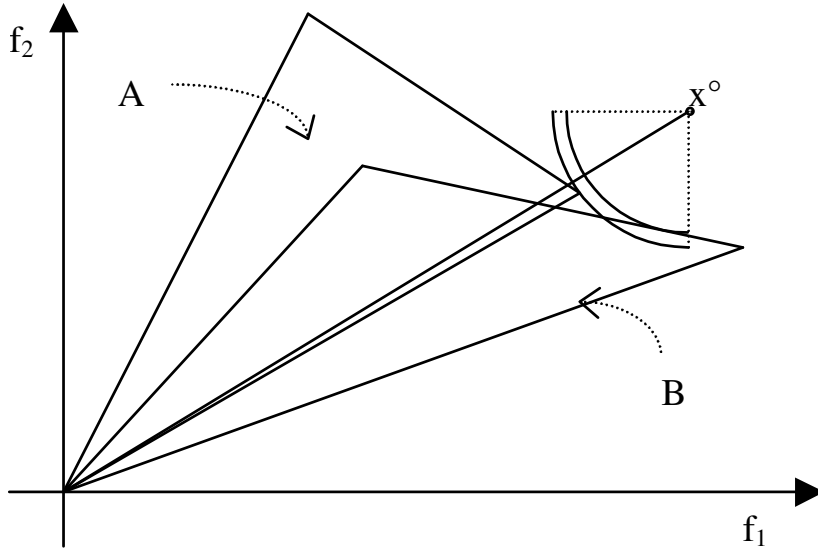


Figure 2: comparison of two capability sets A and B in terms of a deprivation-gap ordering

Figure 2 depicts the minimal $t \in \mathbb{R}_+$ for two capability sets A and B when $x^0 \notin A \cup B$.

Define the following *deprivation-gap ordering*:

$$\text{For all } A, B \in \mathcal{K}, A \succeq^{r^*} B \Leftrightarrow r^*(A) \geq r^*(B).$$

Consider the following axioms:

Definition 5.1. \succeq over \mathcal{K} satisfies

- (5.1.1) **Strong Betweenness** iff, for all $A, B \in \mathcal{K}$, if $A \succ B$, then there exists $C \in \mathcal{K}$ such that B lies entirely in C and $A \succ C \succ B$.
- (5.1.2) **Regressive Domination** iff, for all $A, B \in \mathcal{K}$, if there exists $t > 0$ such that $O(x^0, t) \cap A \neq \emptyset$, and $B \cap O(x^0, t) = \emptyset$, then $A \succ B$.

Strong Betweenness requires that if the standard of living offered by a capability set A is higher than the standard of living offered by a capability set B , then there always exists a capability set C which has B lying entirely in it and which offers a standard of living between A and B . Given that the notion of a capability set B lying entirely in a capability set A is stronger than the notion of B lying entirely in A relative to x^0 , it is straightforward to check that Strong Betweenness implies Betweenness. Regressive Domination is the counterpart of Domination in terms of Uncertain Development, and deals with the situation in which there is a possibility of “regressive development”: if the capability set A dominates the capability set B in the fashion of “regressive development”, then A offers a higher standard of living than B . It can be checked that Regressive Domination implies that, whenever $x^0 \in A$ while $x^0 \notin B$, we must have $A \succ B$. Thus, Regressive Domination is a stronger requirement than Dominance proposed in Section 3.

Theorem 5.2. Suppose \succeq over \mathcal{K} is an ordering. Then, \succeq satisfies Monotonicity, Strong Betweenness, Domination in Terms of Uncertain Development and Regressive Domination if and only if $\succeq = \succeq^{r^*}$.

Proof. We first note that \succeq^{r^*} satisfies Monotonicity, Strong Betweenness, Domination in terms of Uncertain Development and Regressive Domination. Therefore, we need to show if \succeq satisfies Monotonicity, Strong Betweenness, Domination in Terms of Uncertain Development and Regressive Domination, then $\succeq = \succeq^{r^*}$.

Let \succeq be an ordering that satisfies the four properties specified in Theorem 5.2. Note that, since Strong Betweenness implies Betweenness, and

Regressive Domination implies Dominance, from the proof of Theorem 4.1, the following must be true:

- (*) for all $A, B \in \mathcal{K}$, if $x^0 \in A$ and $x^0 \notin B$, then $A \succ B$, and if $x^0 \in A \cap B$, then $r^*(A) \geq r^*(B) \Leftrightarrow A \succeq B$.

Therefore, it remains to be shown that, if $x^0 \notin A \cup B$, then $r^*(A) \geq r^*(B) \Leftrightarrow A \succeq B$.

Let $A, B \in \mathcal{K}$ be such that $x^0 \notin A \cup B$ and $r^*(A) = r^*(B)$. Clearly, $r^*(A) < 0$ since A is closed, compact, star-shaped, and $x^0 \notin A$. For such A and B , we need to show that $A \sim B$. Suppose to the contrary that $A \succ B$ or $B \succ A$. If $A \succ B$, by Strong Betweenness, there exists $C \in \mathcal{K}$ such that B lies entirely in C and $A \succ C \succ B$. Note that, since B lies entirely in C , there exists a positive number $t < -r^*(A)$ such that $O(x^0, t) \cap C \neq \emptyset$. Since $-r^*(A) > t$, it must be the case that $A \cap O(x^0, t) = \emptyset$. By Regressive Domination, $C \succ A$, a contradiction. Similarly, $B \succ A$ leads to a similar contradiction. Therefore, $A \sim B$.

Next, for all $A, B \in \mathcal{K}$, if A, B are such that $x^0 \notin A \cup B$, $r^*(A) > r^*(B)$, then $A \succ B$ follows directly from Regressive Domination.

Therefore, for all $A, B \in \mathcal{K}$, if $x^0 \notin A \cup B$, then $A \succeq B \Leftrightarrow r^*(A) \geq r^*(B)$. This, together with (*), proves Theorem 5.2. ■

6 Concluding Remarks

In this paper we have proposed a new way to measure the standard of living and to compare the standards of living of two persons or, more importantly, two countries. The basis for our approach is Sen's proposal to consider functioning vectors and capability sets. The human development index constructs a real number for each country under investigation. Comparisons among different countries are done by calculating numerical differences of their respective development index. The issue of determining the appropriate weights for those components that enter a development index is central for constructing this index. These weights can and will change over time. Consequently, the aggregate real number will vary under different weighting schemes. In this paper, comparisons among different countries are based on a reference functioning vector that will change as time progresses. The different functionings that constitute this reference vector undergo a re-evaluation over time, also in relation to each other. Functionings are the focus of attention in many

investigations on human development these days. Therefore, we think that the approach formulated here can be used for real-world applications. It should be interesting to see how the rankings according to the currently used HDI would fare in comparison with our new measures.

To conclude this paper, two remarks are in order. First, due to our assumption of the linear “production technology” in producing functioning vectors, we have focused on capability sets that are compact, convex and star-shaped. We realize that the linear “production technology” is a restrictive assumption. It would, therefore, be interesting to relax this assumption and examine the problem of ranking capability sets thus obtained in terms of standards of living offered. Secondly, it is implicitly assumed, given the uncertainty associated with the development of society, that directions along which the agent’s or the country’s functioning vector will grow as time progresses are “equally likely”. One may argue that, even though it is not possible to know the precise direction along which the agent’s functioning vector will grow as time goes by, a range of possible directions can be identified and this range is much narrower than the range of all possible directions implicitly assumed in our framework. It would then be interesting to explore ways of measuring the standard of living offered by capability sets when the information about the range of possible growth directions becomes available. But we leave this point for another occasion.

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